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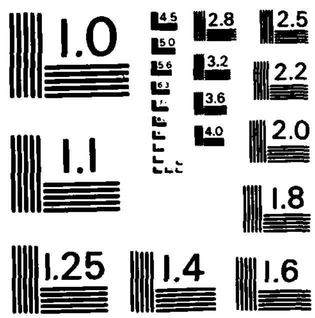

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FINAL SCIENTIFIC REPORT

Air Force Office of Scientific Research
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Period: October 1, 1979 -- April 30, 1983

Title of Research: STABILITY ANALYSIS FOR DIFFERENCE SCHEMES, PROBLEMS IN APPLIED LINEAR ALGEBRA, AND APPLICATION OF NUMBER THEORY TO COMPUTING.

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PROBLEMS IN STABILITY ANALYSIS OF FINITE DIFFERENCE SCHEMES FOR HYPERBOLIC SYSTEMS AND RELATED TOPICS

by

Moshe Goldberg

ABSTRACT

Research completed under Grant AFOSR-79-0127 consists mainly of the following topics: (1) Convenient stability criteria for finite difference approximations to hyperbolic initial-boundary value problems: theory and applications. (2) Operator norms, matrix norms, and multiplicativity. (3) Generalizations of the Perron-Frobenius Theorem and localization of eigenvalues with maximal absolute value.

PROBLEMS IN STABILITY ANALYSIS OF FINITE DIFFERENCE SCHEMES FOR HYPERBOLIC SYSTEMS AND RELATED TOPICS

by

Moshe Goldberg

The purpose of this final scientific report is to summarize my Air Force sponsored research in stability analysis of finite difference approximations of hyperbolic partial differential systems and related topics, during the period October 1979 - April 1983.

1. Convenient Stability Criteria for Difference Schemes of Hyperbolic Initial-Boundary Value Problems.

Consider the first order system of hyperbolic partial differential equations

$$\partial u(x,t)/\partial t = A\partial u(x,t)/\partial x + Bu(x,t) + f(x,t), \quad x \geqq 0, \quad t \geqq 0,$$

where $u(x,t)$ is the unknown vector; A a Hermitian matrix of the form $A^{I} \oplus A^{II}$, $A^{I} < 0, A^{II} > 0$; and $f(x,t)$ is a given vector. The problem is well posed in $L^2(0,\infty)$ if initial values

$$u(x,t) = g(x), \quad x \geqq 0,$$

and boundary conditions

$$u^{I}(0,t) = Su^{II}(0,t) + h(t), \quad t \geqq 0,$$

are prescribed. Here u^{I} and u^{II} are the inflow and outflow unknowns corresponding to the partition of A, and S is a coupling matrix.

In the past years, E. Tadmor and I [12, 13] have succeeded in obtaining easily checkable stability criteria for the above initial-boundary value problem, where the difference approximation consists of an arbitrary basic scheme - explicit or implicit, dissipative or unitary, two-level or multi-level - and boundary conditions of a rather general type.

The first step in our stability analysis was to prove that the approximation is stable if and only if the scalar outflow components of its principal part are stable. This reduced the global stability question to that of a scalar, homogeneous outflow problem of the form

$$\partial u(x,t)/\partial t = a\partial u(x,t)/\partial x, \quad a > 0, \quad x \geqq 0, \quad t \geqq 0,$$
$$u(x,0) = g(x), \quad x \geqq 0; \quad u(0,t) \; 0, \quad t \geqq 0.$$

Investigating the stability of the reduced problem, our main results were restricted to the case where the boundary conditions are translatory, i.e., determined at all boundary points by the same coefficients. This, however, is not much of a limitation since such boundary conditions are commonly used in practice; and in particular, when the numerical boundary consists of a single point, the boundary conditions are translatory by definition.

Our main stability criteria for the translatory case were given essentially in terms of the boundary conditions. Such scheme-independent criteria eliminate the need to analyze the intricate and often complicated interaction between the basic scheme and the boundary conditions; hence providing convenient alternatives to the well known stability criteria of Kreiss [20] and of Gustafsson, Kreiss and Sundström [17].

In our analysis we assumed that the basic scheme is stable for the pure Cauchy problem and that the approximation is solvable. Under these basic assumptions - which are obviously necessary for stability - we found that if the basic scheme is dissipative, then the reduced problem is stable if the boundary conditions are solvable and satisfy the von Neumann condition as well as an additional simple inequality. If the basic scheme is unitary, then instead of satisfying the von Neumann condition, we required that the boundary conditions be dissipative.

Having the new stability criteria, we studied several examples. First, we reestablished the known fact that if the basic scheme is two-level and dissipative, then outflow boundary conditions generated by horizontal extrapolation always maintain stability. Surprisingly, we showed that this result is false if the basic scheme is of more than two levels. Next, for arbitrary dissipative basic schemes we found that if the outflow boundary conditions are generated, for example, by oblique extrapolation, by the Box-scheme, or by the right-sided Euler scheme, then overall stability is assured. Finally, for general basic schemes (dissipative or unitary) we showed that overall stability holds if the outflow boundary conditions are determined by the right-sided explicit or implicit Euler schemes. These examples incorporate and generalize many special cases discussed in recent literature such as [2, 3, 12, 16, 17, 19, 21, 27, 28, 30] and others.

In the past two summers, Tadmor and I [15] were working in order to extend our stability criteria to include a wider range of examples. The proposed new criteria will depend on both the basic scheme and the boundary conditions, but not on the interaction between the two; hence we expect the new results to be as convenient as our scheme-independent criteria in [12, 13]. A first draft of this work is anticipated in the summer of 1983.

Such contributions should be helpful to engineers and applied mathematicians in better understanding and exploiting old and new finite difference approximations to hyperbolic systems.

2. Operator Norms, Matrix Norms, and Multiplicativity.

In the past three years, E.G. Straus and I [7, 8] have continued our study of sub-multiplicative norms and seminorms on operator algebras - an important subject in almost every field of numerical analysis and other areas of applied mathematics. In our work we studied an arbitrary normed vector space $\underset{\sim}{V}$ over the complex field $\underset{\sim}{C}$, with an algebra $\underset{\sim}{B}(\underset{\sim}{V})$ of linear operators on $\underset{\sim}{V}$, and a seminorm N on $\underset{\sim}{B}(\underset{\sim}{V})$. If N is positive definite, i.e., $N(A) > 0$ for all $A \neq 0$, then we call N a generalized operator norm. If in addition, N is (sub-) multiplicative, namely $N(AB) \leq N(A)N(B)$ for all $A,B \in \underset{\sim}{B}(\underset{\sim}{V})$, then N is called an operator norm on $\underset{\sim}{B}(\underset{\sim}{V})$.

Given a seminorm N on $\underset{\sim}{B}(\underset{\sim}{V})$ and a fixed constant $\mu > 0$, then obviously $N_\mu \equiv \mu N$ is a seminorm too. Similarly, N_μ is a generalized operator norm if and only if N is. In both cases, N_μ may or may not be multiplicative. If it is, we say that μ is a multiplicativity factor for N.

Having these definitions we proved in [7] the following:

(i) If N is a nontrivial seminorm or a generalized operator norm on $\underset{\sim}{B}(\underset{\sim}{V})$, then N has multiplicativity factors if and only if

$$\mu_N \equiv \sup\{N(AB) : A,B \in \underset{\sim}{B}(\underset{\sim}{V});\ N(A) = N(B) = 1\} < \infty.$$

(ii) If $\mu_N < \infty$, then μ is a multiplicativity factor for N if and only if $\mu \geq \mu_N$.

Special attention was given by us to the finite dimensional case where it suffices, of course, to consider $\underset{\sim}{C}_{n\times n}$, the algebra of $n \times n$ complex matrices. Following Ostrowski [25], we adopt in this case the terms generalized matrix norm and matrix norm instead of generalized operator norm and operator norm, respectively. We proved in this case that while nontrivial, indefinite seminorms on $\underset{\sim}{C}_{n\times n}$ never have multiplicativity factors, generalized matrix norms always have such factors. In the infinite dimensional case, however, the situation was less decisive, i.e., there exist nontrivial indefinite seminorms and generalized operators on $\underset{\sim}{B}(\underset{\sim}{V})$ which may and may not have multiplicativity factors.

In both the finite and infinite-dimensional cases we proved that if M and N are seminorms on $\underset{\sim}{B}(\underset{\sim}{V})$ such that M is multiplicative, and if $\eta \geqq \zeta > 0$ are constants satisfying

$$\zeta M(A) \leqq N(A) \leqq \eta M(A) \quad \forall \quad A \in \underset{\sim}{B}(\underset{\sim}{V})$$

then any μ with $\mu \geqq \eta/\zeta^2$ is a multiplicativity factor for N.

Using this practical result we showed, for example, that if $\underset{\sim}{V}$ is an arbitrary Hilbert space and

$$r(A) \equiv \sup\{|(Ax,x)| : x \in \underset{\sim}{V}, \quad \|x\| = 1\}, \quad A \in \underset{\sim}{B}(\underset{\sim}{V}),$$

is the classical numerical radius, then μr is an operator norm if and only if $\mu \geqq 4$. This assertion is of interest since the numerical radius r is perhaps the best known nonmultiplicative generalized operator norm [1, 4, 18], and it plays an important role in stability analysis of finite difference schemes for multi-space-dimensional hyperbolic initial-value problems [14, 22, 23, 29].

Our next step was to investigate C-numerical radii which constitute a generalization of the classical radius r, defined by us in [5] as follows: For given matrices $A, C \in \underset{\sim}{C}_{n\times n}$, the C-numerical radius of A is

$$r_C(A) = \max\{|\operatorname{tr}(CU^*AU)| : U \ n\times n \text{ unitary}\}.$$

We have shown [5] (compare [24]) that r_C is a norm on $\underset{\sim}{C}_{n\times n}$ -- and so has multiplicativity factors -- if and only if C is not a scalar matrix and $\operatorname{tr} C \neq 0$. Such multiplicativity factors for the above r_C were found in [5-9].

Our most recent effort in this area was to obtain multiplicativity factors for the well known ℓ_p-norms $(1 \leqq p \leqq \infty)$:

$$|A|_p = \{\Sigma_{i,j}|a_{i,j}|^p\}^{1/p}, \quad A = (a_{i,j}) \in C_{n\times n}.$$

It was shown by Ostrowski [25] that these norms are multiplicative if and only if $1 \leqq p \leqq 2$. For $p \geqq 2$ we have shown [10] that μ is a multiplicativity factor for $|A|_p$ if and only if $\mu \geqq n^{1-2/p}$; thus, in particular, obtaining the useful result that $n^{1-2/p}|A|_p$ is a multiplicative norm on $\underset{\sim}{C}_{n\times n}$.

3. Generalizations of the Perron-Frobenius Theorem and Localization of Eigenvalues with Maximal Absolute Value.

In many instances one is interested in localizing an eigenvalue of maximal absolute value for a given matrix. The most famous result in this vein is the Perron-Frobenius Theorem which states that a matrix with nonnegative elements has at least one nonnegative eigenvalue of maximal absolute value.

Last summer, E.G. Straus and I were looking for generalizations of this celebrated theorem that locate an eigenvalue of maximal absolute value within a certain angle of the complex plane depending on the angle which contains the elements of the matrix. More precisely, let $A_n(\alpha)$ denote the family of all $n \times n$ complex matrices whose entries are contained in a sector

$$S(\alpha) = \{z : |\arg z| \leqq \alpha; \quad 0 \leqq \alpha \leqq \pi \quad \text{fixed}\}.$$

For each $A \in A_n(\alpha)$ let $\beta(A)$ denote the minimal (nonnegative) angle for which the sector $S(\beta)$ contains an eigenvalue of A with maximal absolute value. Thus, defining

$$\beta_n(\alpha) = \sup\{\beta(A) : A \in A_n(\alpha)\}$$

we posed the problem of finding $\beta_n(\alpha)$ as a function of α and n.

Since the Perron-Frobenius Theorem states that $\beta_n(0) = 0$, a wishful generalization would read $\beta_n(\alpha) = \alpha$. This unfortunately is not so, as shown by us in [11] where we give a complete description of the 2×2 case as well as partial results for $n \geqq 3$.

For $n = 2$, for example, we have

$$\beta_2(\alpha) = \begin{cases} \alpha & \text{for} \quad \alpha \leqq \pi/4 \\ \alpha + \pi/2 & \text{for} \quad \pi/4 < \alpha \leqq \pi/2 \\ \pi & \text{for} \quad \alpha > \pi/2, \end{cases}$$

where the discontinuity in $\beta_2(\alpha)$ is typical for all n. More expected properties of $\beta_n(\alpha)$ are:

(i) $\beta_n(\alpha) \geqq \alpha$ for al α an' ..

(ii) $\beta_n(\alpha)$ is a nondecreasing function of α and n.

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NUMERICAL RANGES, EIGENVALUES, MATRIX INEQUALITIES, AND TENSORS

by

Marvin Marcus

ABSTRACT

The results obtained fal. into three separate but related areas: (1) Structure of the numerical range and the localization of eigenvalues. (2) Classical matrix inequalities. (3) Tensors and multilinear algebra.

NUMERICAL RANGES, EIGENVALUES, MATRIX INEQUALITIES, AND TENSORS

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The results obtained fall into three separate but related areas:

(1) Structure of the numerical range and the localization of eigenvalues.

(2) Classical matrix inequalities.

(3) Tensors and multilinear algebra.

The work on the numerical range and eigenvalue localization theory appears in [1, 2, 3, 6, 7, 8, 11, 12]. The numerical range of a linear operator A is the image of the unit sphere under the mapping

$$x \to (Ax, x)$$

It is a classical result of Hausdorff and Toeplitz that the numerical range is a convex region in the plane which contains all the eigenvalues of A. As such, it is of fundamental importance in investigations of stability of difference schemes for partial differential equations. This has been pointed out and developed by a number of authors, including M. Goldberg, one of the investigators on this grant.

In [1], a characterization of unitary operators in terms of a generalized numerical range is obtained. This is then used to analyze the structure of linear operators on spaces of matrices which preserve the numerical range. This work extended earlier results of V.J. Pellegrini [Studia Math. 54 (1975), 143-147].

Items [2] and [3] discuss the structure of the numerical range of a class of sparse 0,1 matrices. Numerical experiments describing the structure of higher numerical ranges are included.

In [6], it is shown that the nonprincipal subdeterminants of a normal matrix satisfy certain quadratic identities. These identities are used to obtain upper bounds on such subdeterminants in terms of elementary symmetric functions of the moduli of the eigenvalues. The same analysis yields lower bounds on the spread of a normal matrix and on the Hilbert norm of an arbitrary matrix. The results in [7] continue these investigations. The size of a nonprincipal subdeterminant is related to the extent to which its main diagonal overlaps the main diagonal of the containing matrix.

In [8], the numerical range is generalized to several maps by defining

$$W(A_1,\ldots,A_k)$$

to be the set of points in the complex plane obtained by multiplying the quadratic forms

$$(A_i x_i,\ x_i),\quad i = 1,\ldots,k$$

where $x_1,\ldots,x_k$ are orthonormal vectors.

The paper [11] contains three elementary proofs of the difficult theorem of Goldberg and Straus on numerical radii (M. Goldberg and E.G. Straus, Linear Algebra and Appl. 24 (1979), 113-131).

In [12], a generalization of the numerical range is defined as follows: Let $1 \leqq r \leqq n$, A a linear transformation on an n-dimensional unitary space V and let

$$G = [G_1:G_2:G_3]$$

be an $r \times 3r$ matrix. A collection of $2r$ vectors

$$x_1,\ldots,x_r,y_1,\ldots,y_r$$

in V is said to be a set of G vectors whenever

$$G_1 = [(x_i,x_j)], \quad G_2 = [(y_i,y_j)], \quad G_3 = [(x_i,y_j)].$$

In [12], the structure of the set $W(A;G)$, of values of the form

$$(Ax_1,y_1) + \cdots + (Ax_r,y_r)$$

where

$$x_1,\ldots,x_r,\ldots,y_r$$

run over all G-vectors is investigated. Specifically, conditions for $W(A;G)$ to be convex, the origin, or empty are given along with some upper bounds on the maximum modulus of any element in $W(A;G)$. These results extend some of the classical results of Hausdorff, Toeplitz, von Neumann, Fan, Berger, and Westwick concerning higher numerical ranges.

The work on classical matrix inequalities appears in [4, 9, 15]. In a research problem (Notices Amer. Math. Soc. 25(7):506 (1978)), A. Abian posed a problem related to the classical Cauchy-Schwarz Inequality. Let V be a unitary space and let A, B, P, Q be linear on V. The question is: what are necessary and sufficient conditions that

$$(Av,u)(Bu,v) \leqq (Pu,u)(Qv,v)$$

for all u,v in V? In [4], this question is completely resolved. In [9], the same problem is reexamined in a more general setting: Let A, B, P, Q be n-square nonsingular complex matrices, and for $1 \leqq m < n$

let U and V be $n \times m$ complex matrices. For $n \geqq 3$ necessary and sufficient conditions are given for the inequality

$$\det(U^*AV)\det(V^*BU) \leqq \det(U^*PU)\det(V^*QV)$$

to hold for all U and V.

The paper [15] gives an elementary proof of an inequality for any principal subdeterminant of a positive-definite Hermitian matrix A. In particular, let C be a k-square principal submatrix of the inverse of A. Let D be the inverse of the principal submatrix of A lying in the same numbered rows and columns that define D. Then $C \geqq D$ where the inequality sign denotes dominance in the sense that $C - D$ is positive semi-definite Hermitian.

Items [5, 14, 16] are concerned with tensor spaces and multilinear algebra. Certain linear groups of operators can be defined in terms of generalized matrix functions. These groups are characterized in [5]. This work is related to earlier work of G.M. de Oliveira and J.A. Dias da Silva.

In [14], an index is defined for spaces of tensors. This index is computed for the Grassmann space, the tensor space, and the completely symmetric space.

In [16], a detailed analysis of the equality of decomposable symmetrized tensors is described. This work extended earlier work of Freese, Merris, Pierce, and Williamson.

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17. Interior points of the generalized numerical range (with M. Sandy), in press.

18. Bilinear ranges, convexity and elementary doubly stochastic matrices (with M. Sandy and K. Kidman), in press.

19. Unitarily invariant norms, condition numbers and the Hadamard product (with M. Sandy and K. Kidman), in progress.

ELEMENTARY DIVISORS, GENERAL INEQUALITIES, HADAMARD MATRICES, AND PERMANENTS

by

Henryk Minc

ABSTRACT

My work sponsored by Air Force Grant AFOSR-79-0127 was concerned with the following topics: (1) Inverse elementary divisor problems for nonnegative matrices. (2) Bounds for permanents. (3) *Inequalities*. (4) Hadamard matrices. (5) Theory of permanents and its applications. (6) The van der Waerden permanent conjecture. (7) Minimum of the permanent of a doubly stochastic matrix with prescribed zero entries.

ELEMENTARY DIVISORS, GENERAL INEQUALITIES, HADAMARD MATRICES, AND PERMANENTS

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1. Inverse Elementary Divisor Problems for Nonnegative Matrices.

One of the most important unsolved problems in linear algebra is the inverse eigenvalue problem for nonnegative matrices: to find necessary and sufficient conditions that a given n-tuple of complex numbers be the spectrum of a nonnegative matrix. A parallel problem for doubly stochastic matrices in unsolved as well. The inverse elementary divisor problem for doubly stochastic matrices, the determination of necessary and sufficient conditions that given polynomials be the elementary divisors of a doubly stochastic matrix, contains the inverse eigenvalue problem, and obviously it is also unsolved.

In [4] it is shown that for any real α, $-\frac{1}{n-1} < \alpha < 1$, and any positive integers $e_2,\ldots,e_m$ whose sum is $n-1$, there exist doubly stochastic $n \times n$ matrices with elementary divisors $\lambda-1$ and $(\lambda-\alpha)^{e_i}$, $i = 2,\ldots,m$. This result implies that for any $n \geqq 3$ there exist doubly stochastic $n \times n$ matrices which have no roots. In [4] the inverse elementary divisor problem is considered for doubly stochastic matrices modulo the inverse eigenvalue problem: given a doubly stochastic matrix, does there exist a doubly stochastic matrix with the same spectrum and arbitrarily prescribed elementary divisors consistent with the spectrum that do not include $(\lambda-1)^k$ with $k > 1$ (otherwise the answer would clearly be in the negative). The question is answered in [4] in the negative in general, and in the affirmative in case the given matrix is positive, diagonalizable, and with real eigenvalues.

In [5] it is proved that given any positive diagonalizable matrix, there exists a positive matrix with the same spectrum and with any prescribed elementary divisors consistent with the spectrum. A parallel result for doubly stochastic matrices was also proved, thus extending the result in [4] to diagonalizable positive doubly stochastic matrices with complex, not necessarily real, eigenvalues.

2. Bounds for Permanents.

In [2], bounds for permanents of real matrices are obtained. In [1] Friedland's lower bound for the permanents of doubly stochastic matrices is utilized to obtain an improved lower bound for the d-dimensional dimer problem for $d \geqq 4$:

$$\lambda_d \sim \tfrac{1}{2} \log(2d) - \tfrac{1}{2}.$$

For the all important 3-dimensional case it is known that

$$0.418347 \leqq \lambda_3 < 0.548271,$$

where the lower bound is due to Hemmersley and the upper bound is due to Minc. In order to improve these bounds it is necessary to obtain sharper bounds than the currently known bounds for the permanents of (0,1)-circulants with 6 ones in each row. For this purpose, permanents of some 850 (0,1)-circulants were computed, of orders up to 18×18, with 3, 4 or 6 ones in each row. No definite results have been obtained so far.

3. Inequalities.

Let $(a_{1j}, a_{2j}, \ldots, a_{m_j j})$ be non-negative m_j-tuples, $j = 1,\ldots,n$, where $m_1 \geqq \ldots \geqq m_n \geqq 1$, and $\Sigma_{j=1}^{n} m_j = M$. Let

$$\alpha'_{11} \leqq \alpha'_{21} \leqq \ldots \leqq \alpha'_{m_1 1} \leqq \alpha'_{12} \leqq \alpha'_{22} \leqq \ldots \leqq \alpha'_{m_2 2} \leqq \ldots \leqq \alpha'_{1n} \leqq \alpha'_{2n} \leqq \ldots \leqq \alpha'_{m_n n}$$

be the M numbers a_{ij}, $i = 1,\ldots,m_j$, $j = 1,\ldots,n$, arranged in non-decreasing order, and

$$\alpha^*_{11} \geqq \alpha^*_{21} \geqq \ldots \geqq \alpha^*_{m_1 1} \geqq \alpha^*_{12} \geqq \alpha^*_{22} \geqq \ldots \geqq \alpha^*_{m_2 2} \geqq \ldots \geqq \alpha^*_{1n} \geqq \alpha^*_{2n} \geqq \ldots \geqq \alpha^*_{m_n n}$$

be the same numbers arranged in non-decreasing order.

In [3], inequalities of the following types are proven. If $a_{ij} \leqq 1$, $i = 1,\ldots,m_j$, $j = 1,\ldots,n$, then

$$\sum_{j=1}^{n} \prod_{i=1}^{m_j} \alpha_{ij} \leqq \sum_{j=1}^{n} \prod_{i=1}^{m_j} \alpha'_{ij}.$$

and

$$\prod_{j=1}^{n} (1 + a_{1_j} a_{2_j} \ldots a_{m_j j}) \leqq \prod_{j=1}^{n} (1 + \alpha'_{1_j} \alpha'_{2} \ldots \alpha'_{m_j j}).$$

If $a_{ij} \geqq 1$, $i = 1,\ldots,m_j$, $j = 1,\ldots,n$, then

$$\sum_{j=1}^{n} \prod_{i=1}^{m_j} \alpha_{ij} \leqq \sum_{j=1}^{n} \prod_{i=1}^{m_j} \alpha^*_{ij}$$

and

$$\prod_{j=1}^{n} (1 + a_{1j} a_{2j} \ldots a_{m_j j}) \leqq \prod_{j=1}^{n} (1 + \alpha^*_{1j} \alpha^*_{2j} \ldots \alpha^*_{m_j j}).$$

In either case

$$\prod_{j=1}^{n} \sum_{i=1}^{m_j} a_{ij} \geqq \prod_{j=1}^{n} \sum_{i=1}^{m_j} \alpha^*_{ij}.$$

Some of these inequalities were used to establish bounds for permanents of nonnegative matrices.

4. Hadamard Matrices.

Let $\Delta(n)$ be the maximum of the absolute value of the determinant of $n \times n$ $(1, -1)$-matrices. Fréchet asked if there exists a simple analytic expression for $\Delta(n)$ as a function of n, and he proposed the problem of determining an analytic asymptotic expression for $\Delta(n)$. It is known that $\Delta(n) \leqq n^{n/2}$, and that equality, for $n > 2$, can hold only if $n \equiv 0 \bmod 4$. In [7] relevant known results for all n are surveyed, and it is concluded that Fréchet's question should be answered in the negative, although

$$\log \Delta(n) \sim \frac{n}{2} \log n.$$

5. Theory of Permanents and Its Applications.

The last five years witnessed an increased research activity in the theory of permanents and its applications. At least 75 new research papers on the subject and similar publications were written or were actually published during the period 1978-1981; this represents nearly 20% of the total literature on permanents in the last 170 years.

In [9] the developments in the area are surveyed. The paper includes a detailed discussion of Egorycev's proof of the van der Waerden conjecture (see below), a section on bounds of permanents of $(0,1)$-matrices, and on their applications to the dimer problem, to the problem of enumeration of Latin rectangles, and other topics. The paper contains a report on the current status of each of the 20 conjectures and 10 problems, which were listed as unsolved in Minc's "Permanents" (1978). Additional 10 conjectures and 3 problems are proposed.

6. The Van der Waerden Permanent Conjecture.

Research in the theory of permanents and its applications in the last two years has been strongly influenced by the recent solutions of the van der Waderden permanent conjecture by Egoryčev and by Falikman. They both proved that

$$\operatorname{per}(S) \geqq n!/n^n \tag{1}$$

for any $n \times n$ doubly stochastic matrix S. In addition, Egoryčev showed that equality can hold in (1) if and only if S is the $n \times n$ matrix all of whose entries are $1/n$. In paper [9] a version of Egoryčev's proof is given. In [6] two alternative variants of his proof are presented. In [8] versions of Egoryčev's proof and of Falikman's proof are given in detail.

7. Minimum of the Permanent of a Doubly Stochastic Matrix with Prescribed Zero Entries.

Paper [10] is a study of properties of matrices with minimum permanent in a face of Ω_n, the polyhedron of doubly stochastic $n \times n$ matrices, i.e., for doubly stochastic matrices with zero entries in prescribed fixed positions. Egoryčev proved that all permanental cofactors of a matrix with minimal permanent in Ω_n are equal. This implies that in such a matrix any pair of rows (columns) can be replaced by their mean without change in permanent. This averaging process leads to a proof of the van der Waerden conjecture. Unfortunately, in the case of doubly stochastic matrices with prescribed zero entries such an averaging method has only a restricted application (viz., to rows (columns) with same prescribed zero pattern). In [10] it is proved that permanental cofactors of a matrix with minimal permanent in a face of Ω_n cannot exceed the permanent of the matrix; and that the permanental cofactors of entries which are not prescribed, are actually all equal to the permanent of the matrix. This result is then used to obtain minimum permanents in faces of Ω_n in which all prescribed zeros are restricted to two rows or columns, or in which the prescribed zeros form a submatrix.

In some cases in which prescribed zeros are located in many rows (columns), Falikman's method seems to be more appropriate than that of Egoryčev. Some partial results, as yet unpublished, have been obtained.

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Henryk Minc
October 1979 - April 1983

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8. The van der Waerden permanent conjecture, General Inequalities 3 (to appear).

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10. Minimum of the permanent of a doubly stochastic matrix with prescribed zero entries. Linear and Multilinear Algebra, to appear.

APPLICATIONS OF NUMBER THEORY TO COMPUTATION

by

Morris Newman

ABSTRACT

Work on the Air Force Project was entirely concerned with the application of number theory to high speed digital computation, using as tools residue arithmetic, p-adic arithmetic, and multiprecision arithmetic.

APPLICATIONS OF NUMBER THEORY TO COMPUTATION

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Work on the Air Force project was entirely concerned with the application of number theory to high speed digital computation, using as tools residue arithmetic, p-adic arithmetic, and multiprecision arithmetic. Numerous programs for the now defunct ILLIAC4 were prepared which took advantage of the parallel processing features of this machine. In particular, programs were prepared using residual arithmetic for the following:

(1) The exact solution of an integral system of linear equations, and the exact computation of the determinant of the system.

(2) The determination of the exact inverse of an integral matrix using minimal storage.

(3) The determination of the rank and a basis for the null space of an integral matrix;

(4) The determination of all rational solutions of an integral system of linear equations.

(5) The determination of the eigenvalues of a rational symmetric triple diagonal matrix to any desired accuracy.

(6) The computation of the permanent of a matrix.

(7) The determination of the Hermite normal form of an integral matrix.

(8) The determination of the exact characteristic polynomial of an integral matrix.

(9) The determination of the Smith normal form of an integral matrix.

In addition, a program was prepared (for a serial machine) which finds the exact solution of a linear system by p-adic arithmetic, rather than residue arithmetic. The advantage is that only one prime modulus is necessary, and the numerous local solutions modulo differing primes are replaced by simple matrix by vector multiplications modulo the single prime. In additionl the Chinese Remainder Theorem is <u>not</u> required; only some rather simple multiprecision multiplications, divisions, and additions need be performed. The time required to find the exact solution in this way compares very favorably with the time required to find an ordinary solution.

Finally, work on this project resulted in three Master's theses supervised by M. Newman.

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Morris Newman
October 1979 - April 1983

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SINGULAR VALUES, INVARIANT FACTORS, THE MATRIX EXPONENTIAL, AND EIGENVALUES

by

Robert C. Thompson

ABSTRACT

The results achieved centered around the properties of singular values, of invariant factors of matrices, of the exponential function acting on matrices, and of eigenvalues of finite matrices. Continuing study progress has been made on a wide class of matrix questions, some of which are self contained, and others lead to some of the most fundamental aspects of linear algebra.

SINGULAR VALUES, INVARIANT FACTORS,
THE MATRIX EXPONENTIAL, AND EIGENVALUES

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The results achieved centered around the properties of singular values of invariant factors of matrices, of the exponential function acting on matrices and of eigenvalues of finite matrices, with all these topics and some others subsumed under the general theme of inequalities in linear algebra. Some typical results are cited below. Because of the many results achieved, only representative ones can be mentioned here.

1. Properties of Singular Values.

Some of the deepest questions in linear algebra fall under this heading, it being realized during the review period that the proper approach to them is through the study of the Lie algebras and Lie groups. The efforts to develop this approach during the review period will yield many results in future years.

Items [1], [4], [7], [10], [14], [16], and [18] present new results on singular values. Only [7] will be described here. In [7] a new type of numerical range for matrices was defined, and a complete analysis of it is given. Let A be a fixed matrix. The numerical range in question is the principal diagonal of all matrices UAU^t as U ranges over all unitary matrices (t denotes transpose). The complete characterization found for this numerical range is given in terms of the singular values of A.

2. Properties of Invariant Factors.

These are the invariant factors that appear in the Smith canonical form for integer or polynomial matrices. They were investigated in a variety of ways, in [2], [3], [5], [6], [8], [9], [12], [17], [24], [25], and [26]. In [6] conditions were given on how invariant factors behave when matrices add, and these conditions were further investigated in [5], [25], and [17]. The conditions were a set of divisibility relations, and [25] attempted to discuss the sufficiency of these conditions. The difficulties encountered are analyzed in [17]. In [9] the existence and uniqueness of invariant factors was discussed using a new method.

In [12] some of the author's previous work on the eigenvalues of sums of Hermitian matrices was utilized to give a short proof of an elegant inequality pertaining to the behavior of invariant factors under matrix multiplication. This inequality is just one of a large class found by the author in a previous review period, but never published. The merit of [12] is the simple proof it gives of one inequality from this large class.

3. The Matrix Exponential.

A key, and rather fundamental, conjecture was formulated, namely, if A and B are Hermitian matrices, then unitary matrices U and V exist such that $e^{iA}e^{iB} = e^{i(UAU^{-1}+VBV^{-1})}$. A number of results concerning this conjecture appear in [11], [20], [21], and [17]. Partial proofs are known for the conjecture, and it is known how to formulate it in Lie-theoretic terms. If A and B are not Hermitian, the same conjecture can be formulated (with U, V now just nonsingular, not necessarily unitary). It has been proved (in [20]) that this latter conjecture can be valid only for matrices A, B sufficiently near zero. The conjecture is important because it bears on the relationship between a Lie group and its Lie algebra.

4. Properties of eigenvalues.

Paper [16] investigated the relationship between the diagonal elements and the eigenvalues of a normal matrix. It is a long open question to clarify this relationship. Paper [16] showed that a rather natural condition evolving from the author's work on singular values could not give the full answer to this question, even though it yielded infinitely many constraints.

5. Embedding Properties.

Linear algebra is full of results determining when one matrix can be embedded in another. There is a large amount of already existing research on this theme under the heading of unitary dilation theory. Paper [10] showed that, in spite of the impressive quantity of unitary dilation results in the functional analysis literature, there are many as yet untouched ramifications in related directions. Specifically, it was shown how dilations to doubly stochastic matrices, to unimodular (integer) matrices, to complex orthogonal matrices, instead of to unitary matrices, lead to many questions, some of which can be solved. Although [10] gave many results, its significance is the hints it gives for future directions of research.

6. Matrix Inequalities.

One of the most fundamental inequalities in mathematics is the triangle inequality, and in an earlier period it was found how to extend it to a matrix valued version: Given matrices A and B, there exist unitary matrices U and V such that $|A+B| \leq U|A|U^{-1} + V|B|V^{-1}$, where $|\cdot|$ denotes a matrix valued absolute value acting on matrices. Further investigation of this inequality was carried out in [14] and [1]. There is related number theoretical research to be reported in [19]. Multiplicative versions of the matrix triangle inequality have also been investigated, with partial success.

PUBLICATIONS

Robert C. Thompson
October 1979 - April 1983

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22. Products of simple involutions, 15 typed pages.

23. A supplement to the Campbell-Baker-Hausdorff formula, 6 typed pages.

24. Left multiples and right divisors of integral matrices, 11 typed pages

25. Sums of integral matrices, 17 typed pages.

26. Matrices over rings of algebraic integers, 24 typed pages.

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